

Registration No.:

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Total Number of Pages: 02

Course: M.Sc.I
Sub_Code: FMCC901

9th Semester Regular Examination: 2024-25

SUBJECT: Functional Analysis

BRANCH(S): M.Sc.I(MC)

Time: 3 Hours

Max Marks: 70

Q.Code: R012

Answer Question No.1 (Part-I) which is compulsory, any five from rest (Part-II)
The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions: (2 x 10)

- Is every Banach space a Hilbert space? Justify your answer through a counter example.
- Define spectrum and point spectrum.
- Is the mapping $R^2 \rightarrow R^2$ defined by $(x_1, x_2) \rightarrow (x_1, 0)$ an open mapping?
- Define Hahn-Banach theorem.
- Show that every orthonormal sets are linearly independent.
- Suppose X is a finite dimensional normed space. What can you say about the dimension of the dual of X ?
- State Riesz representation theorem for $C([a, b])$.
- Give an example of a self-adjoint operator on K^2 .
- When every linear operator on X will be bounded?
- Give an example of an unbounded operator.

Part-II

Long Answer Type Questions (Answer Any five)

- Q2 a) Show that every closed and bounded subset of normed space X is compact. (5+5)
b) State and prove uniform boundedness principle.
- Q3 a) State and prove open mapping theorem. (5+5)
b) For $K = \mathbb{R}/\mathbb{C}$ and X, Y be normed space over K , and $T : X \rightarrow Y$ be linear. Then the following are equivalent: (i) T is bounded. (ii) T is continuous.
- Q4 a) Show that the space l^p , $p \neq 2$ is not an inner product space. (5+5)
b) Find the orthonormal basis of $P_3[-1, 1]$ starting from the basis $1, n, n^2, n^3$. Use the inner product defined by $f \cdot g = \int_{-1}^1 f(t) \cdot g(t) dt$.

- Q5** a) State and prove Riesz representation theorem of functionals. (5+5)
b) Establish a linear isometry from $L^p([a, b])$ into the dual of $L^q([a, b])$, where $1 \leq p \leq \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$.
- Q6** a) Let H be a finite dimensional Hilbert space over K and A is a bounded linear operator on H . Suppose that $K = \mathbb{C}$ and A is normal or $K = \mathbb{R}$ and A is self-adjoint. Then prove that there exists an orthonormal basis for H consisting of eigenvectors of A . (5+5)
b) write the properties of normal, self-adjoint, and unitary operator.
- Q7** a) Show that the spectrum $\sigma(T)$ of a bounded self adjoint linear operator $T: H \rightarrow H$ on a complex Hilbert space H is real. (5+5)
b) Prove or disprove that a unitary operator is closed.
- Q8** a) If A is a bounded linear operator on a Hilbert space and A_n is a compact operator for every positive inter n such that $\|A_n - A\|$ converges to 0, then prove that A is compact. (5+5)
b) If X and Y be two Banach spaces and $f: X \rightarrow Y$ be a closed linear map then show that f is continuous.